What Do Rock/Paper/Scissors and Baseball Have in Common?

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Notes provide additional information and were reminders during the presentation. They are not supposed to be anything close to a complete text of the presentation or thorough discussion of the subject.

Use Acrobat Reader's ability to enlarge what appears on the screen if you have trouble reading a graph or table.

Game Theory Applies

- Simple solution for Rock/Paper/Scissors
  - If opponent detects pattern, can take advantage
  - Play each choice randomly 1/3 of the time
  - Example of a "mixed strategy" – not always doing same thing
  - Opponent can't gain from knowing your strategy
- Mathematical Game Theory:
  - Optimal strategy is usually a mixed one
  - Opponent can't gain from knowing your optimal strategy
- Baseball is far more complicated than R/P/S
  - Will focus on bunting with runner on first, none outs
  - Pitchers excluded
  - Data: Retrosheet, 2004-08 major league play, both leagues

Mathematical game theory can be used to study both and other games. R/P/S is very simple to analyze, but baseball is quite complex and there are many opportunities such as trying to steal or hit and run while the defense may decide to pitch out or not.

For two person games like these, the optimal solution is usually a mixed strategy—not making the same choice on every play and making the choices at random according to the solution probabilities.

Retrosheet pitch and bunt data are best since 2004, so used those five years of data.

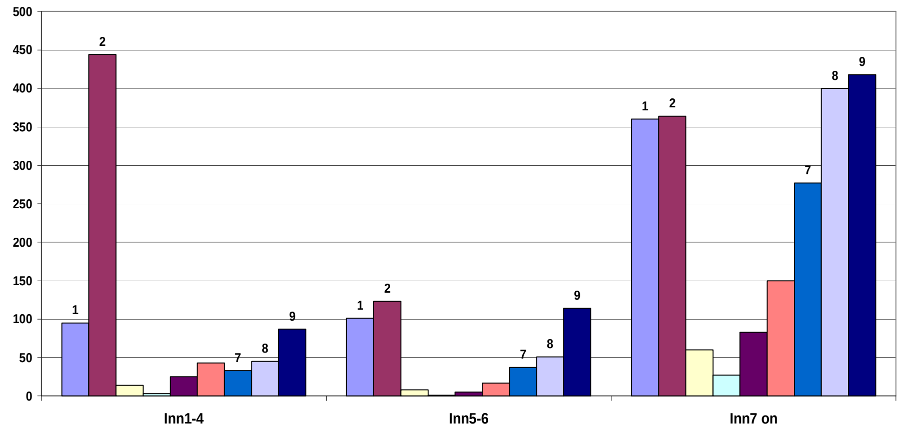
Potential Bunt Situations

- Runner on first, no outs
- Score is "close"
  - No more than 2 runs down
  - In 9th or later, not down more than one run
  - Ahead by 3 or fewer
- Will analyze by groups of innings
  - Early: 1-4; bunt not expected by defense
  - Middle: 5 and 6
  - Late: 7 and later; bunt likely expected

Runner on first and none out is by far the most common situation where bunting may be done. Want the game to be fairly close, and a different definition than the one shown could be used.

Key to the analysis will be the assumption, which is often but not always the case, that the defense is not expecting a bunt in the early innings and is expecting one and defending accordingly in the late innings. The middle innings are in between as the graphs that follow will show.

Who bunts how often? (by lineup position)

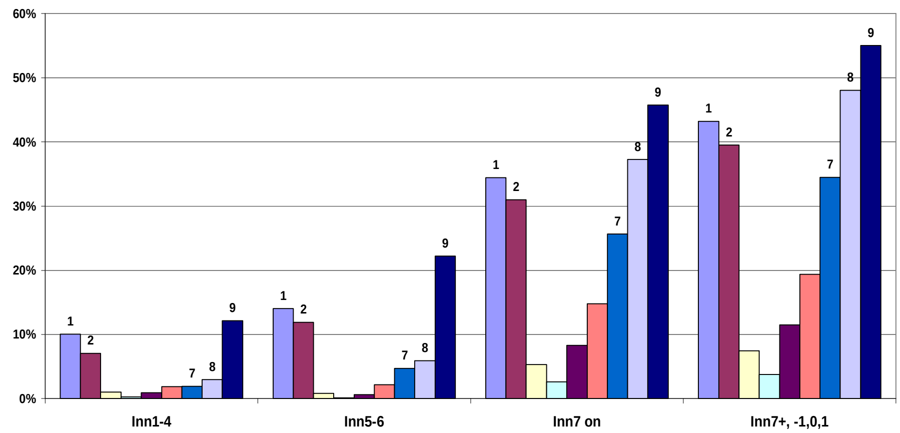


The frequency of bunting by inning group and lineup position (but pitchers are excluded) are shown, so nine bars in each of the three groups. The #2 hitters with 444 bunts in the early innings clearly have the most bunts for that group. The next graph will show that is primarily because they have the most opportunities.

The 3,4,5 hitters almost never bunt, as would be expected. In the late innings of close games, #1, #7-9 also bunt frequently.

The next graph shows the percentage of the potential bunt situations in which the batters bunt.

Percentage of bunting (by lineup position)



There is a fourth group of nine bars. The one on the right is for the late innings (7 and on) with the score either tied or a one run difference. I'll call these "very close" as opposed to just "close."

We see the #2 bunt percentage in the early innings is less than #1 and #9. Their count is much higher because they naturally get more potential situations in the early innings.

Note that the percentage of bunting increases for all lineup positions as the game progresses, and is somewhat higher in the late and very close situations where the #9 hitters bunt in over half of the potential situations.

These percentages support the assumption that the defense is not expecting a bunt in the early innings and is likely to be expecting one for the 1,2,7,8, and 9 hitters in the late and close situations.

Analyze #2 Lineup Position

- Want comparison of play outcomes:
  - In potential bunting situations
  - Bunt when expected (innings 7 and on)
  - Bunt when not expected (innings 1-4)
  - Hit away when bunt expected (7 and later)
  - Hit away when bunt not expected (1-4)
- Number 2 hitters have the most data

There are several reasons the #2 position is interesting to analyze:

•As Willie Sutton did not say, that's where the data are

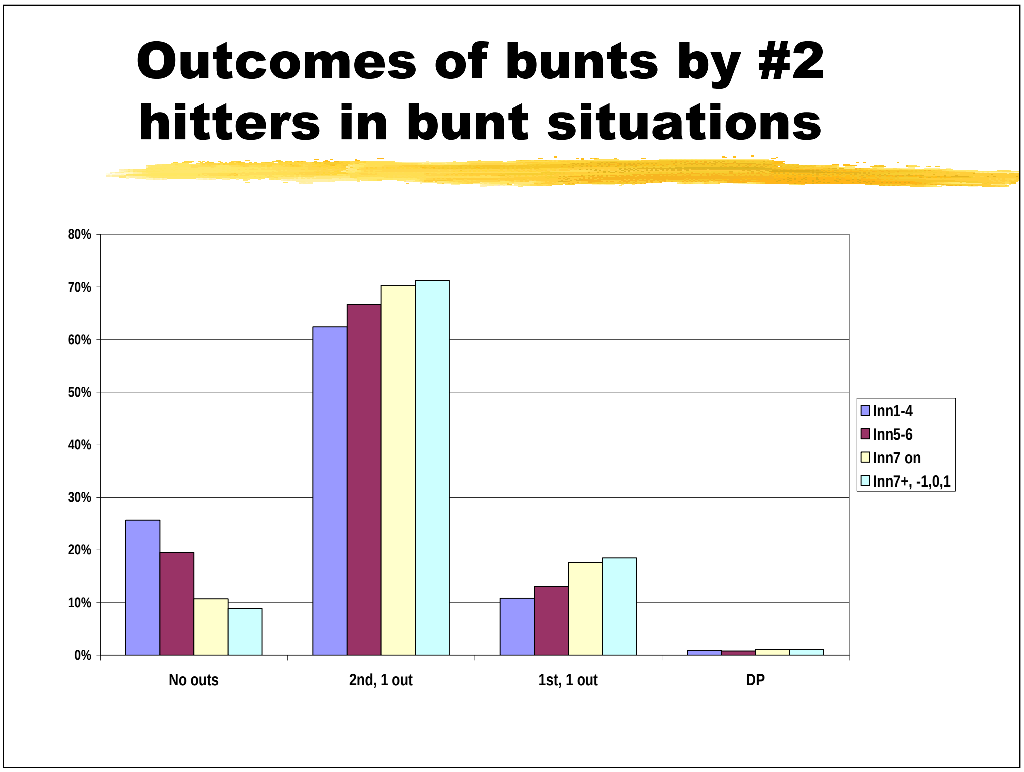
•#2 hitters are usually fairly good ones, so the question of whether or not to bunt can be critical

•The criticality is reinforced by the best hitters in the lineup coming up next

•There are some computational advantages (as we will see later) to not having to worry about the #9 hitter coming up

The four cases will be the basis of the game theoretical analysis shown later.

We will first see charts illustrating the outcomes in these four situations plus a few more



Each group has four bars for the groups of innings: 1-4, 5-6, 7 on, 7 on very close (other cases are close). Each group of bars shows a group of bunt play outcomes.

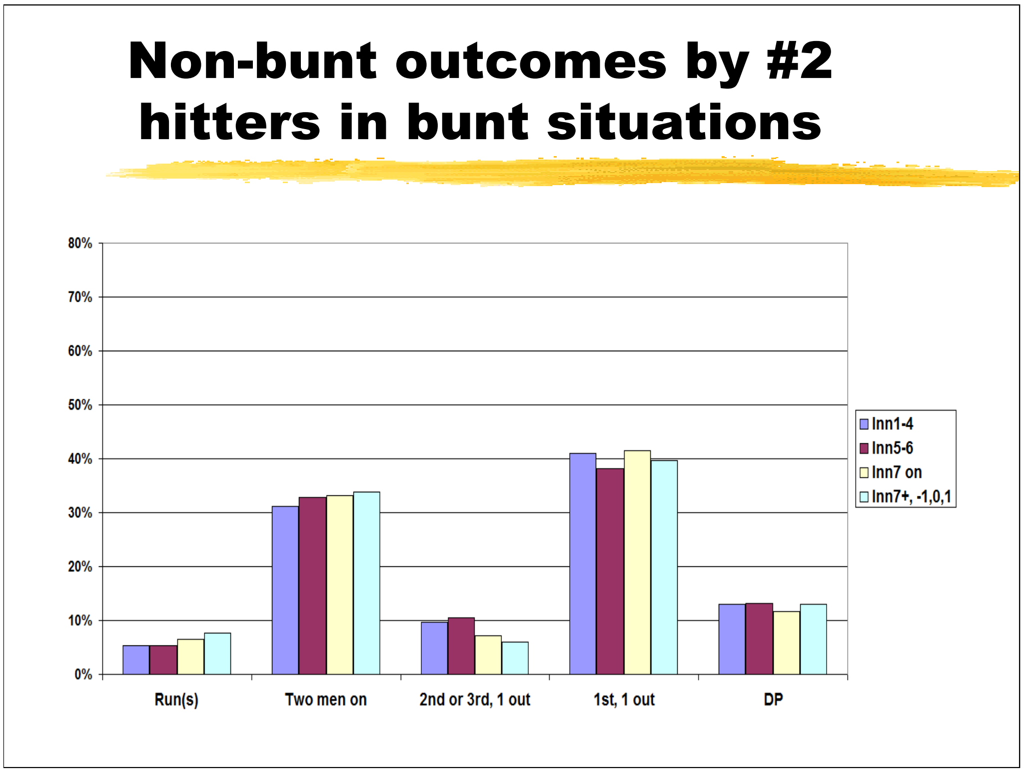
First group ends up with two men on and no outs. Note that is far more likely in the early innings when the bunt is not expected. These are bunt hits or fielding errors, and almost all of them are first and second

Second group is a normal sacrifice with a runner on second and one out.

Third group is failed sacrifice ending with runner on first and one out. These two groups are more likely in the later innings, likely because hits are far less likely.

Fourth group is double plays, which are quite unusual.

Next we see what happens when not bunting.



Similar to previous chart, but with somewhat different groups of four bars. Leftmost is when at least a run scores, which does not happen with bunts. That means homer or extra base hit or rarely, single + error. Note becomes more likely in later innings when defense is likely to be looking for bunt.

Next group has two men on, so no outs on play. Also a little more likely in later innings.

Next group has runner on 2nd (most of cases) or 3rd and an out. Less likely when looking for bunt.

Next to last group is runner on first and an out, and last group is a double play. Both of these do not show a trend as the game progresses.

Note that on this and the previous slide, there is very little difference in the late innings between close and very close.

Quantitative Analysis

- Use combined 2004-08 hitting data for lineup positions 3 through 8
- Compute chances of scoring at least 1, 2, or 3 runs in remainder of inning
  - Possible objectives to score or prevent
  - Starting with #2 hitter and one runner on and none out, by time #9 hitter might be up:
    - inning will be over with 0, 1, or 2 runs, or
    - at least 3 runs will have scored

Data from Retrosheet by lineup position tracks batting by all players, starters or replacements, who hit in each lineup spot. Advantage to not having to worry about #9, which will be different between leagues. St. Louis often bats pitcher #8, but that won't have much of an effect on the analysis since we are starting with the #2 hitter.

Markov model is used to compute the probabilities of scoring at least one, two, or three runs. These are possible objectives a manager may want to achieve or prevent.

Payoff Matrix – R/P/S

		Player 2's Choice		
		Rock	Paper	Scissors
Player 1's Choice	Rock	0	-1	1
	Paper	1	0	-1
	Scissors	-1	1	0

- Values are payoff from Player 2 to 1
- Player 1 wants to maximize
- Player 2 wants to minimize

A two person game can be represented by a payoff matrix showing each player’s possible choices for each play. The one for Rock/Paper/Scissors is an example. If both players use the optimal mixed strategy, the long term payoff, the “value” of the game will be zero.

Payoff Matrix: Score at least one run

		Defense Choice: Defend bunt? Yes	No
Offense Choice	Bunt	44.9%	49.1%
	Hit Away	46.8%	45.4%

- Probabilities of scoring >=1 run
- Offense wants to maximize
- Defense wants to minimize
- No obvious "pure strategy"
- Mixed strategies are needed

Here is the payoff matrix for the objective of scoring at least one run in the remainder of the inning. The values shown, from the Markov model, are the probabilities of scoring.

The matrix is simplified to only two choices due to the data available for the analysis. In a real game the offense has additional choices (rows in the matrix) such as trying to steal or a hit and run play. The defense has additional choices (columns in the matrix) such as deciding to pitch out or possibly the type of pitch to throw. In this sense, each pitch is another play of the game.

Since each side can improve if it knows the other side's choice, there is no single choice, a "pure strategy" that is optimal, so the solution is a mixed strategy for each side, which we will see in the next slide.

Payoff Matrix: Score at least one run

		Defense Choice	
		Defend bunt?	
		Yes	No
Offense Choice	Bunt	44.9%	49.1%
	Hit Away	46.8%	45.4%

Optimal Strategies for scoring/preventing >=1 run

Offense	Bunt: 65%	Hit Away: 35%
Defense	Defend: 25%	Don't: 75%
If both sides use these strategies: 46.3%		

The calculations are not particularly difficult, but are not shown. For the offense, bunting 65% and for the defense defending against the bunt 25% of the time are the optimal choices. With them, the value of the game is a 46.3% chance of scoring at least one run. Note that this is in the middle of each row and each column. However, the difference between the four values in the matrix is small, so in a sense, it doesn't matter that much what choices the teams make. Specific game conditions such as the batter, following batters, pitcher and possible relief pitchers are likely to be more important than these results based on average players and results.

Payoff Matrix: Score at least two runs

		Defense Choice	
		Defend bunt?	
		Yes	No
Offense Choice	Bunt	23.3%	27.2%
	Hit Away	27.7%	27.0%

Optimal Strategies for scoring/preventing >1 run

Offense	Bunt: 4%	Hit Away: 96%
Defense	Defend: 16%	Don't: 84%
If both sides use these strategies:	**27.0%**	

The payoff matrix with the probabilities of scoring at least two runs in the rest of the inning shows that bunting is not a particularly effective way to do this. The optimal strategies show bunts and defending against them should not be done very often. The optimal value looks the same as the value for hitting away and not defending against the bunt, but it is slightly higher if more decimal places are shown.

Payoff Matrix: Score at least three runs

		Defense Choice	
		Defend bunt?	
		Yes	No
Offense Choice	Bunt	11.2%	14.4%
	Hit Away	15.6%	15.0%

- ⌘Not bunting is best choice
- ⌘So defense won’t play for bunt

The second row “dominates” the first row, so offense should never bunt if wanting to score at least three runs, which is hardly a surprise. Knowing that, the defense will never defend against the bunt if it wants to minimize the chances of three or more runs scoring.

Other considerations

- Game theory:
  - Choices chosen not knowing opponent’s
  - Offense likely can tell what defense is doing
  - Defense may have a good idea if bunt or not
  - Teams can try to vary or disguise choice
- Managers’ objectives may be different
- After #2 hits, analysis does not allow for
  - Defense: pitching changes, intentional walks
  - Offense: pinch hitters, pinch runners

One problem with the analysis is that game theory presumes that both players make their choices without knowing the other one’s. In the potential bunt situation, this is not strictly the case. The positioning of the infielders normally reveals what the defense is expecting. However, teams will sometimes try to disguise what they are doing by moving the infielders as the pitch is being delivered. Similarly, the batter may show bunt and then decide to hit away or vice-versa.

One complexity not analyzed is that the two managers may have different objectives. For example, if the score is tied in the top of the 7th, the offense may feel it is critical to score and go ahead while the defense may think it has good hitters coming up and the batting team’s bullpen is weak, so it may want to keep the game close and prevent two or more runs from scoring.

Conclusions

- Most studies: bunts hurt scoring chances
- Game theory: bunting more may be good
  - Gain in probability of scoring may be small
  - #2 hitters bunted 40% in 7+, tied, +/- 1 run
  - "Optimal" called for 65% for scoring >=1
  - Analysis dependent on several assumptions
  - Other moves (steal, hit and run) not considered
  - Game specifics may be more important
  - Helpful not to be predictable

Most prior analysis has shown that bunting, except by very weak hitters such as pitchers, is not a good play either from trying to score at least one run or to increase total runs scored. The game theoretic analysis using real data shows that bunting some of the time can be a way to increase the chances of scoring at least one run.

A prior chart showed that #2 hitters bunted in about 40% of the potential situations in the late innings with the score tied or at most a one run difference. This is below the optimum 65% in the analysis based on average play. Possibly, late inning bunting in very close games should be done more, particularly if it looks like the defense is not expecting a bunt. However, considerations such as the bunting ability of the #2 hitter, a sacrifice followed by an intentional walk of Pujols or a very strong hitter may come into play.

Web sites, e-mail

www.pankin.com/baseball.htm

has details about Markov model (used to compute scoring probabilities) and other baseball studies

E-mail: mp --ATsign-- pankin.com

Plan to post slides, notes on my web site